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A joint pricing and inventory control problem under an energy buy-back program

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ABSTRACT

The demand for power keeps rising with rapid economic development and growth of industrialization. The frequent mismatch created between demand and supply can be mitigated by the use of energy buy-back programs. This paper models a buy-back program using a periodic review joint pricing and inventory model, incorporating compensations and setup cost over finite planning horizons. It is shown that an (s, S, A, P^*) policy is optimal for the decision maker for maximizing the expected total profit.

1. Introduction

In recent years, the rapid developments of economies and growth of industrialization have led to excessive consumption of energy and resources. Factors such as slow growth of fuel supply exacerbate the gap between electricity supply and demand. In order to mitigate the electricity supply shortage, a variety of energy buy-back programs are widely adopted; see [8,13,14]. For example, in the US energy market, Wisconsin Electric created an energy buy-back program referred to as power market incentives that pays large industrial customers for voluntarily reducing electric load during peak time; Commonwealth Edison (ComEd) Company created a load response program named ComEd smart returns that pays business customers financial incentives for reducing the electricity usage during times of high demand [1].

Such programs, when activated, provide participating industrial customers with a certain amount of financial compensation for reducing their energy use during peak time, and aim to encourage electricity customers to move their energy consumption from peak time to non-peak time, thus releasing the demand pressure during peak time and smoothing the overall electricity consumption for all time periods. Many high energy consumption manufacturers actively participate in various energy buy-back programs. However, such participation will influence their pricing and inventory decisions since they have to deal with the balance between (i) receiving financial compensation as a result of participation by reducing production and (ii) wishing to increase sales and satisfy customers' demands, through production/inventory control.

Chen et al. [5] consider a periodic review production/inventory problem in which a manufacturer participates in such an energy buy-back program that rewards participants with a certain amount of financial compensation for reducing energy use. It is shown that a base-stock policy is optimal for the non-peak market condition, whereas the (s, S) policy is optimal for peak market conditions. Chao and Chen [2] consider a similar problem with continuous time production/inventory setting in two cases, whose settings are both formulated as Markovian decision processes. It is shown that in the first case of the exponential peak-period duration, the production and shutdown strategy is determined by a single threshold level, whereas in the second case when the peak duration becomes known at the beginning of a peak period, the strategy is determined by a sequence of threshold levels depending on the remaining time before the current peak period ends. One simplification of such models is the exclusion of setup cost, which seems to be fairly common in real-world practice.

Because of the advances in information technology and the development of E-commerce, a dynamic pricing strategy has been adopted by a plethora retailing and manufacturing companies to improve their management and performance. It has become an effective tool for managing demands and reducing the production and inventory pressure; see for example [3,6,9–11,15]. Specifically, Federgruen and Heching [9] develop a combined pricing and inventory control model under uncertainty without an energy buy-back program. With the assumption that the ordering cost is proportional to the order quantity, it is shown that a base-stock–list-price policy is optimal. In this policy, the optimal

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replenishment policy in each period is characterized by an order-up-to level, and the optimal price depends on the initial inventory level at the beginning of each period. Besides, the optimal price is a nondecreasing function of the initial inventory level. Chen and Simchi-Levi [6,7] further extend the model proposed in [9] with the inclusion of a fixed cost and show that the profit-to-go function is symmetric k-concave when the demand function is in the general form, and thus an (s, S, A, P^*) policy is optimal. Chao et al. [3] consider a dynamic inventory and pricing optimization problem in a periodic review inventory system with setup cost and finite ordering capacity in each period. With strong CK-concavity, it is shown that the optimal inventory policy can be partially characterized by an (s, s', p) policy. Zhu [15] analyzes the combined pricing and inventory control problem in a random and price-sensitive demand environment with return and expediting, and shows that the optimal replenishment policy is a modified base-stock policy, the pricing policy is a modified base-stock–list-price policy, and the optimal policy for inventory adjustment follows a dual-threshold policy. Chen et al. [4] study a multi-period inventory planning problem in which the firm can source from two possibly unreliable suppliers for a price-dependent demand, and characterizes the optimal procurement policy and the conditions under which the optimal policy reveals a monotone response to changes in the inventory level.

Taking the approaches based primarily on [6,5], we consider a joint pricing and inventory model under an energy buy-back program over finite T planning horizons, in which the compensations, setup cost and general demand function are incorporated. At the beginning of each period, pricing and inventory decisions are simultaneously made. The objective is to identify the optimal joint pricing and inventory strategy under an energy buy-back program in order to maximize the expected total profit. It is shown that, for the general situation with an energy buy-back program, the optimal joint pricing and inventory strategy is also of (s, S, A, P^*) type.

The rest of this paper is organized as follows. The next section introduces the model formulation. Section 3 presents the main result. Section 4 provides a numerical example and the final section concludes this paper.

2. Settings and the model

Consider a manufacturer who has to make joint pricing and inventory decisions over finite T planning horizons. Assume that the demands in different periods are independent of each other and all shortages are backlogged.

At period t, suppose there are M peak states and define a financial compensation, L_{ti}, for each peak state i, with $L_{ti} > 0$, $i = 1, 2, \ldots, M$, corresponding to the energy buy-back level. Without loss of generality, we can get these L_{ti}s sorted, namely, $0 < L_{t1} \le L_{t2} \le \cdots \le L_{tM}$. For each period t, we introduce a constant setup cost $K > 0$, and let c_t be the unit variable cost, w_t be the demand, P_t, $\underline{P}_t$ and $\overline{P}_t$ be the selling price, and lowest and highest feasible selling price, respectively, $P_t \in [\underline{P}_t, \overline{P}_t]$, x_t and y_t be inventory levels before and after production, respectively, $y_t \ge x_t$, and p_{ti} be the probability for peak state i to happen. Furthermore, we have the following assumptions for the compensation levels and demand function.

Assumption 1. For period t, $L_{t1} \ge \overline{L}_{t+1} = \sum_{j=1}^{M} p_{(t+1)j} L_{(t+1)j}$.

This assumption can be realized when the p_{tj}, $j = 2, \ldots, M$, are relatively small; see [5,12].

Assumption 2. For period t, the demand function w_t satisfies

$$w_t = D_t(P, \varepsilon_t) := \alpha_t D_t(P) + \beta_t$$

where α_t and β_t are random variables with $\mathrm{E}\{\alpha_t\} = 1$ and $\mathrm{E}\{\beta_t\} = 0$. The random perturbations, $\varepsilon_t = (\alpha_t, \beta_t)$, are independent across time; $D_t(P, \varepsilon_t)$ is non-increasing and concave in $P \in [\underline{P}_t, \overline{P}_t]$; and the expected demand $d_t = \mathrm{E}\{w_t\} = D_t(P)$ is a finite and strictly decreasing function in $P \in [\underline{P}_t, \overline{P}_t]$.

The additive case and multiplicative case that can be considered as special cases of $D_t(P, \varepsilon_t)$ are both common in the economics literature [11].

Assumption 3. For period t, $D_t^{-1}(d)$, the inverse function of $D_t(P)$, is continuous and strictly decreasing. The expected revenue, $R_t(d) := dD_t^{-1}(d)$, is a concave function.

For period t, state i, the production cost includes both a fixed cost and a variable cost:

$$\delta(y_t - x_t)(L_{ti} + K) - L_{ti} + c_t(y_t - x_t)$$
$$\text{where } \delta(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0. \end{cases}$$

Due to backlogging, x_t may be positive or negative. A cost $h_t(x)$ is incurred at the end of period t that represents the inventory holding cost when $x > 0$, and the penalty cost when $x < 0$. Furthermore, let $G_t(y, P) = \mathrm{E}\{h_t(y - D_t(P, \varepsilon_t))\}$. For $h_t(x)$ and $G_t(y, P)$, we have the following two assumptions similar to those in [6,9].

Assumption 4. For period t, $h_t(x)$ is a convex function of the inventory level x and

$$\lim_{y\to\infty} G_t(y, P) = \lim_{y\to-\infty} [c_t y + G_t(y, P)] = \lim_{y\to\infty} [(c_t - c_{t+1})y + G_t(y, P)] = \infty.$$

Assumption 5. For period t, $0 \le G_t(y, P) = O(|y|^\rho)$ for some integer ρ, and $\mathrm{E}\{D_t(P, \varepsilon_t)\}^\rho < \infty$ for all $P \in [\underline{P}_t, \overline{P}_t]$.

On the basis of the above assumptions, the expected total profit Π over finite T planning horizons is

$$\Pi = \mathrm{E}\left\{\sum_{t=1}^{T}\left\{R_t(d_t) - G_t(y_t, P_t) - c_t(y_t - x_t) - [\delta(y_t - x_t)(L_{ti} + K) - L_{ti}]\right\}\right\}.$$

At the beginning of period t, state i, the decision maker has to decide whether to stop producing, and participate in the energy buy-back program, or to start producing, and choose an inventory level y_t after production and selling price P_t, in order to maximize the expected total profit Π over finite T planning horizons.

According to Assumption 2, there is a one-to-one correspondence between the selling price P_t and the expected demand $\mathrm{E}\{w_t\} = D_t(P_t) = d_t \in [\underline{d}_t, \overline{d}_t]$ where $\underline{d}_t = D_t(\overline{P}_t)$ and $\overline{d}_t = D_t(\underline{P}_t)$.

Therefore, the problem can be formulated as one of selecting, at period t, state i, an inventory level y_t and an expected demand level d_t such that the expected total profit Π' over finite T planning horizons is maximized:

$$\Pi' = \mathrm{E}\left\{\sum_{t=1}^{T}\left\{R_t(d_t) - G_t(y_t, d_t) - c_t(y_t - x_t) - [\delta(y_t - x_t)(L_{ti} + K) - L_{ti}]\right\}\right\}. \quad (1)$$

Denote by $v_t(x, i)$ the profit-to-go function at the beginning of period t, state i, with an initial inventory level x, and let $v_{T+1}(x, j) \equiv$

0 for $j = 1, \ldots, M$. Then for period t, we have the following dynamic program for Π':

$$v_t(x,i) = \max_{y\geq x}\Big\{R_t(d_t(y)) - G_t(y,d_t(y)) - c_t(y-x) - [\delta(y-x) \times (L_{ti}+K) - L_{ti}] + \mathrm{E}\{v_{t+1}(y-\alpha_t d_t(y) - \beta_t, j)\}\Big\}$$

where $d_t(y)$ is the expected demand associated with the best selling price, P, for a given inventory level y at period t. By letting

$$F_t(x) = \sum_{j=1}^{M} p_{(t+1)j} v_{t+1}(x,j),$$

$$g_t(y,d) = R_t(d) - c_t y - G_t(y,d) + \mathrm{E}\{F_t(y-\alpha_t d - \beta_t)\}$$

the dynamic program can be transformed to

$$v_t(x,i) = c_t x + \max_{y\geq x}\Big\{-[\delta(y-x)(L_{ti}+K) - L_{ti}] + g_t[y, d_t(y)]\Big\}. \quad (2)$$

3. Analysis

We use a more general demand function definition in this paper and thus need to utilize a weaker version of k-convexity, referred to as symmetric k-convexity, developed in [6].

Definition. A function $f(x)$ is called sym-k-convex for $k \geq 0$ if for any x_1, x_2 and $\lambda \in [0, 1]$,

$$f[(1-\lambda)x_1 + \lambda x_2] \leq (1-\lambda)f(x_1) + \lambda f(x_2) + \max\{\lambda, 1-\lambda\}k.$$

A function $f(x)$ is called sym-k-concave if $-f(x)$ is sym-k-convex.

We summarize the properties of a sym-k-convex function as follows; see [6] for details.

Lemma 1. *Suppose $f(x)$ and $g(x)$ are symmetric k- and m-convex, respectively. We have:* (i) *$f(x)$ is sym-n-convex for any $n \geq k$;* (ii) *$\alpha f(x) + \beta g(x)$ is sym-$(\alpha k + \beta m)$-convex if $\alpha, \beta \geq 0$;* (iii) *if ξ is a random variable such that $\mathrm{E}\{|f(x-\xi)|\} < \infty$ for all x, then $\mathrm{E}\{f(x-\xi)\}$ is sym-k-convex;* (iv) *$f(x) + a$ is sym-k-convex for any constant a;* (v) *if $f(x)$ is continuous and $\lim_{|x|\to\infty} f(x) \to \infty$, then there exists a pair (s, S) with $s \leq S$ such that: $f(S) \leq f(x)$ for all x; s is the smallest value x such that $f(S) + k = f(x)$; $f(x) > f(s)$ for all $x < s$, and $f(x) \leq f(z) + k$ for all x, z with $(s+S)/2 \leq x \leq z$.*

Theorem 1. *For period t, state i,*

(i) *$g_t(y,d) = O(|y|^\rho)$ and $v_t(x,i) = O(|x|^\rho)$; $g_t(y,d)$ is continuous in (y,d) and for any $d \in [\underline{d}_t, \overline{d}_t]$, $\lim_{|y|\to\infty} g_t(y,d) \to -\infty$—hence, for any fixed y, $g_t(y,d)$ has a finite maximizer, denoted by $d_t(y)$;*
(ii) *$g_t(y,d)$ and $v_t(x,i)$ are sym-$(K+L_{ti})$-concave;*
(iii) *there exists a pair (s_t^i, S_t) with $s_t^i \leq S_t$ and a set $A_t^i \subset [s_t^i, (s_t^i + S_t)/2]$ such that it is optimal to: (a) reject the energy buy-back program, set the expected demand level $d_t = d_t(S_t)$, and produce $S_t - x_t$ products when $x_t < s_t^i$ or $x_t \in A_t^i$; or (b) accept the energy buy-back program, set $d_t = d_t(x_t)$ and produce nothing when $x_t \geq s_t^i$ and $x_t \notin A_t^i$.*

Proof. Due to the assumptions and the one-to-one correspondence between d_t and P_t, the proof for (i) is similar to those in [6,9]. We now prove (ii) by induction.

For period T, state i, since $v_{T+1}(x,j) \equiv 0$ for all j, we have

$$g_T(y, d_T(y)) = \max_d\Big\{R_T(d) - c_T y - G_T(y,d) + \mathrm{E}\{F_T(y-\alpha_T d - \beta_T)\}\Big\} = \max_d\Big\{R_T(d) - G_T(y,d)\Big\} - c_T y.$$

First of all, it follows from the convexity of $h_t(x)$ that $G_t(y,d) = \mathrm{E}\{h_t(y-\alpha_t d - \beta_t)\}$ is jointly convex in (y,d); see [6,9] for a proof. Since $R_T(d)$ and $G_T(y,d)$ are concave and convex, respectively, it is easy to verify the concavity of $\max_d\{R_T(d) - G_T(y,d)\}$. Hence, $g_T(y, d_T(y))$ is concave and also sym-$(K+L_{Ti})$-concave. Therefore, there exists a maximizer S_T to $g_T(y, d_T(y))$, and a solution s_T^i with $s_T^i \leq S_T$ to the equation $g_T(y, d_T(y)) = g_T(S_T, d_T(S_T)) - (K + L_{Ti})$. According to (2), we have

$$v_T(x,i) - L_{Ti} - c_T x = \begin{cases} g_T(S_T, d_T(S_T)) - (K+L_{Ti}), & x < s_T^i \\ g_T(x, d_T(x)), & x \geq s_T^i. \end{cases}$$

The $(K+L_{Ti})$-concavity of $v_T(x,i) - L_{Ti} - c_T x$ can be justified from the concavity of $g_T(y, d_T(y))$, and we can conclude the sym-$(K+L_{Ti})$-concavity of $v_T(x,i) = (v_T(x,i) - L_{Ti} - c_T x) + L_{Ti} + c_T x$ according to Lemma 1.

Hence, we conclude that both $g_T(y, d_T(y))$ and $v_T(x,i)$ are sym-$(K+L_{Ti})$-concave at period T.

Now suppose that $v_{t+1}(x,i)$ is sym-$(K + L_{(t+1)i})$-concave for period $t+1$. From Assumption 1, Lemma 1 and the sym-$(K + L_{(t+1)i})$-concavity of $v_{t+1}(x,i)$, it is easy to verify the sym-$(K + \overline{L}_{t+1})$-concavity of F_t. Then for any $y_1 < y_2$ and $\lambda \in [0,1]$, we have

$$F_t((1-\lambda)(y_1 - \alpha_t d_t(y_1) - \beta_t) + \lambda(y_2 - \alpha_t d_t(y_2) - \beta_t)) \geq (1-\lambda)F_t(y_1 - \alpha_t d_t(y_1) - \beta_t) + \lambda F_t(y_2 - \alpha_t d_t(y_2) - \beta_t) - \max\{\lambda, 1-\lambda\}(K + \overline{L}_{t+1}). \quad (3)$$

From the concavity of $R_t(d)$ and $-h_t$, we have

$$R_t((1-\lambda)d_t(y_1) + \lambda d_t(y_2)) \geq (1-\lambda)R_t(d_t(y_1)) + \lambda R_t(d_t(y_2)) \quad (4)$$

$$-h_t((1-\lambda)(y_1 - \alpha_t d_t(y_1) - \beta_t) + \lambda(y_2 - \alpha_t d_t(y_2) - \beta_t)) \geq -(1-\lambda)h_t(y_1 - \alpha_t d_t(y_1) - \beta_t) - \lambda h_t(y_2 - \alpha_t d_t(y_2) - \beta_t). \quad (5)$$

By adding (3) (4) (5) together and taking the expectation, we have

$$g_t((1-\lambda)y_1 + \lambda y_2, (1-\lambda)d_t(y_1) + \lambda d_t(y_2)) \geq (1-\lambda)g_t(y_1, d_t(y_1)) + \lambda g_t(y_2, d_t(y_2)) - \max\{\lambda, 1-\lambda\}(K + \overline{L}_{t+1}). \quad (6)$$

Since $d_t((1-\lambda)y_1 + \lambda y_2)$ maximizes $g_t((1-\lambda)y_1 + \lambda y_2, y)$ for $d \in [\underline{d}_t, \overline{d}_t]$,

$$g_t((1-\lambda)y_1 + \lambda y_2, d_t((1-\lambda)y_1 + \lambda y_2)) \geq g_t((1-\lambda)y_1 + \lambda y_2, (1-\lambda)d_t(y_1) + \lambda d_t(y_2)). \quad (7)$$

With inequalities (6) and (7), we have

$$g_t((1-\lambda)y_1 + \lambda y_2, d_t((1-\lambda)y_1 + \lambda y_2)) \geq (1-\lambda)g_t(y_1, d_t(y_1)) + \lambda g_t(y_2, d_t(y_2)) - \max\{\lambda, 1-\lambda\}(K + \overline{L}_{t+1}).$$

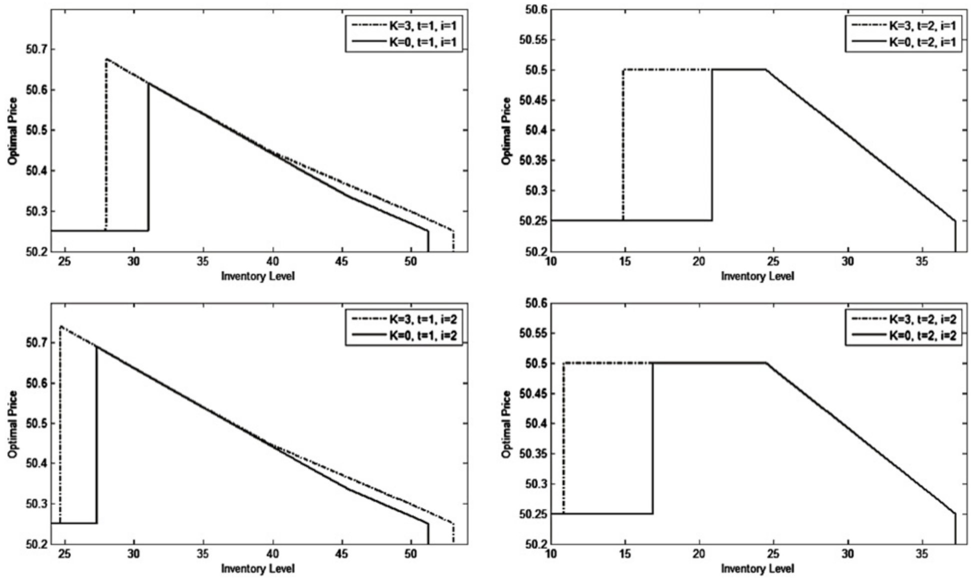

Fig. 1. Optimal prices P^* for varying K as a function of x.

Therefore, we derive the sym-$(K+\bar{L}_{t+1})$-concavity of $g_t(y, d_t(y))$, and consequently the sym-$(K + L_{ti})$-concavity of $g_t(y, d_t(y))$ according to Assumption 1 and Lemma 1.

For period t, state i, define $v_t^*(x, i) = v_t(x, i) - L_{ti} - c_t x$. From (2), the sym-$(K + L_{ti})$-concavity of $g_t(y, d_t(y))$ and Lemma 1, we have

$$\begin{aligned} v_t^*(x, i) &= v_t(x, i) - L_{ti} - c_t x \\ &= \begin{cases} g_t(S_t, d_t(S_t)) - (K + L_{ti}), & x \in I_t \\ g_t(x, d_t(x)), & x \notin I_t \end{cases} \end{aligned} \quad (8)$$

where S_t is the maximizer of $g_t(y, d_t(y))$ and $I_t = \{y \le S_t : g_t(y, d_t(y)) \le g_t(S_t, d_t(S_t)) - (K + L_{ti})\}$. Furthermore, we have

$$v_t^*(x, i) \ge \begin{cases} g_t(x, d_t(x)) & \text{for any } x \\ g_t(S_t, d_t(S_t)) - (K + L_{ti}) & \text{for any } x \le S_t. \end{cases}$$

Let s_t^i be the smallest value y such that $g_t(y, d_t(y)) = g_t(S_t, d_t(S_t)) - (K + L_{ti})$. We then have $(-\infty, s_t^i) \subset I_t$ and $[(s_t^i + S_t)/2, \infty] \subset I_t^c$. To derive the sym-$(K + L_{ti})$-concavity of $v_t(x, i)$, it suffices to prove that $v_t^*(x, i)$ is sym-$(K + L_{ti})$-concave according to Lemma 1. Let $x_\lambda = (1 - \lambda)x_1 + \lambda x_2$ for $x_1 \le x_2, \lambda \in [0, 1]$, and consider four different cases: (a) $x_1, x_2 \notin I_t$; (b) $x_2 \in I_t$; (c) $x_2 \notin I_t, x_1 \in I_t$ and $x_\lambda \le S_t$; and (d) $x_2 \notin I_t, x_1 \in I_t$ and $x_\lambda \ge S_t$. In what follows, we only show that the result holds for case (d) and omit the proofs for the other three cases since they follow a similar logic.

If $x_2 \notin I_t, x_1 \in I_t$ and $x_\lambda \ge S_t$, then there exists $0 \le \mu \le \lambda$ such that $x_\lambda = (1 - \mu)S_t + \mu x_2$, and

$$\begin{aligned} & v_t^*(x_\lambda, i) \\ &= g_t(x_\lambda, d_t(x_\lambda)) \\ &\ge (1 - \mu)g_t(S_t, d_t(S_t)) + \mu g_t(x_2, d_t(x_2)) \\ &\quad - \max\{\mu, 1 - \mu\}(K + L_{ti}) \\ &\ge (1 - \lambda)g_t(S_t, d_t(S_t)) + \lambda g_t(x_2, d_t(x_2)) \\ &\quad + (\lambda - \mu)(g_t(S_t, d_t(S_t)) - g_t(x_2, d_t(x_2))) - (K + L_{ti}) \\ &\ge (1 - \lambda)(-(K + L_{ti}) + g_t(S_t, d_t(S_t))) \\ &\quad + \lambda g_t(x_2, d_t(x_2)) - \lambda(K + L_{ti}) \\ &\ge (1 - \lambda)v_t^*(x_1, i) + \lambda v_t^*(x_2, i) - \max\{\lambda, 1 - \lambda\}(K + L_{ti}). \end{aligned} \quad (9)$$

Table 1
Optimal stock policies for varying K.

	(s_1^1, S_1)	(s_1^2, S_1)	(s_2^1, S_2)	(s_2^2, S_2)
$K = 0$	(31.05, 51.21)	(27.32; 51.21)	(20.88, 37.25)	(16.88, 37.25)
$K = 3$	(28.01, 53.06)	(24.74, 53.06)	(14.88, 37.25)	(10.88, 37.25)

In (9), the first inequality follows from the sym-$(K + L_{ti})$-concavity of $g_t(y, d_t(y))$; the third inequality follows from the fact that S_t is the maximizer of $g_t(y, d_t(y))$ and $\mu \le \lambda$.

Consequently, we can conclude the sym-$(K + L_{ti})$-concavity of $v_t^*(x, i)$, and thereby the sym-$(K + L_{ti})$-concavity of $v_t(x, i)$.

Hence, we conclude that both $g_t(y, d_t(y))$ and $v_t(x, i)$ are sym-$(K + L_{ti})$-concave at period t. The proof for (ii) is complete.

Let $A_t^i = I_t \cap [s_t^i, (s_t^i + S_t)/2]$; (iii) follows from (ii) and Lemma 1. □

Due to the one-to-one correspondence between d_t and P_t, the optimal policy (iii) in Theorem 1 can be transformed to an (s, S, A, P^*) policy.

Corollary 1. *For period t, state i, there exists a pair (s_t^i, S_t) with $s_t^i \le S_t$ and a set $A_t^i \subset [s_t^i, (s_t^i + S_t)/2]$ such that it is optimal to: (a) reject the energy buy-back program, set $P_t^* = D_t^{-1}(d_t(S_t))$, and produce $S_t - x_t$ products when $x_t < s_t^i$ or $x_t \in A_t^i$; or (b) accept the energy buy-back program, set $P_t^* = D_t^{-1}(d_t(x_t))$ and produce nothing when $x_t \ge s_t^i$ and $x_t \notin A_t^i$.*

4. A numerical example

Consider a manufacturer facing a two-period two-state joint pricing and inventory problem. We set two-period two-state compensation levels $L_{11} = 7, L_{12} = 10, L_{21} = 5, L_{22} = 7$ with the two-state probabilities being $p_{21} = 0.9$ and $p_{22} = 0.1$, respectively, for the second period; $\alpha = 1$; $\beta \sim U[-25, 25]$; unit variable cost $c_1 = c_2 = 0.5$; holding and shortage cost function $h_1(x) = h_2(x) = |x|$; expected demand function $d = 100 - p$. The results for the setup costs $K = 0$ and 3 are summarized in Table 1 and Fig. 1.

Table 1 gives the optimal stock policies (s_t^i, S_t) with or without setup cost. It can be observed that S_t increases while s_t^i decreases as K increases from 0 to 3. Fig. 1 shows that the optimal price P^* increases as K increases from 0 to 3. Both results illustrate that the setup cost has obvious impacts on the optimal joint pricing and inventory policy for an energy buy-back problem and, in general, S_t and P^* will increase while s_t^i will decrease as K increases.

5. Concluding remarks

This model can be extended to the infinite horizon expected discount or average profit model with stationary parameters under an energy buy-back program, whose analysis may be more complicated due to the involvement of the convergence of a sequence of finite horizon problems. However, by using the methodology for extending the results for the finite horizon to the infinite horizon given by Chen and Simchi-Levi [7], the main results can be demonstrated to remain the same, along with a solution for the optimality equation of the infinite horizon model for an energy buy-back problem.

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